

Algebraic methods in Combinatorics**HS 08
Exercise Set 1****Exercise 1**

Let A and B be two $n \times n$ matrices over an arbitrary field.

1. Prove that

$$rk(A + B) \leq rk(A) + rk(B). \quad (1)$$

2. Show an example in which inequality (1) is strict.

Exercise 2

Let A be an $n \times n$ matrix with entries in \mathbb{R} and let $x \in \mathbb{R}^n$ be a column vector. Prove that $x^T A^T x = x^T A x$.

Exercise 3

Let G be a graph with vertex set $V = \{1, 2, \dots, n\}$. Let A be the adjacency matrix of G . Prove that, for every $1 \leq i, j \leq n$ and for every positive integer k , the (i, j) th entry of A^k is the number of walks (an edge can be traversed more than once) of length k between i and j in G .

Exercise 4

Let $B_1, B_2, \dots, B_m \subseteq K_n$ be complete bipartite graphs that cover every edge of K_n an odd number of times. Prove that $m \geq (n - 1)/2$.