

***Algebraic Methods in Combinatorics******HS 2008******Exercise Set 12*****Due date:****Exercise 1**

Prove the following special case of the modular Ray-Chaudhuri-Wilson Theorem (with a slightly weaker conclusion, which is still good enough for Borsuk's problem): Let  $p$  be a prime, and let  $\mathcal{F} \subseteq \binom{[n]}{2p-1}$  be such that  $|A \cap B| \neq p-1$  for any  $A, B \in \mathcal{F}$ . Then

$$|\mathcal{F}| \leq \binom{n}{0} + \binom{n}{1} + \dots + \binom{n}{p-1}.$$

*Hint.* To every  $A \in \mathcal{F}$ , assign its characteristic vector  $c_A \in \{0, 1\}^n$  and a function  $f_A : \{0, 1\}^n \rightarrow \mathbb{F}_p$  given by a suitable polynomial (evaluated modulo  $p$ ).

**Exercise 2**

Show that the Shannon capacity of the disjoint union of two graphs satisfies

$$\Theta(G + H) \geq \Theta(G) + \Theta(H).$$