Exercise 1

Let $\gamma_1, \ldots, \gamma_m$ be positive real numbers, and let $\lambda$ be a real number. Let $C = \text{diag}(\gamma_1, \ldots, \gamma_m)$, and let $A = \lambda I_m + C$. Calculate $\det(A)$.

**Hint:** Consider $\det(B)$, where $B$ is a square matrix of order $m + 1$, whose first row is $(1, \ldots, 1)$, and whose $i$th row, for $2 \leq i \leq m + 1$, is the $(i - 1)$st row of $A$ to which a 0 was added in the first entry.

Exercise 2

1. Let $f, g \in \mathbb{R}[x_1, \ldots, x_n]$ be two multilinear polynomials. Prove that if $f(x_1, \ldots, x_n) = g(x_1, \ldots, x_n)$ for every $(x_1, \ldots, x_n) \in \{0, 1\}^n$, then $f = g$ as polynomials (that is, for every $I \subseteq [n]$, the coefficient of $\prod_{i \in I} x_i$ in $f$ is equal to the coefficient of $\prod_{i \in I} x_i$ in $g$).

2. Give an example of a field $F$ and two different polynomials (that is, they differ in at least one coefficient) $f, g \in F[x]$, such that $f(a) = g(a)$ for every $a \in F$.

Exercise 3

In this exercise we prove a (very) special case of Snevily’s Conjecture. Let $K = \{k_1, \ldots, k_r\}$ and let $L = \{l_1, \ldots, l_s\}$. Let $\mathcal{F} = \{F_1, \ldots, F_m\}$ be a family of sets that satisfies the following properties:

1. $|F_i| \in K$ for every $1 \leq i \leq m$.
2. $|F_i \cap F_j| \in L$ for every $1 \leq i < j \leq m$.
3. $\bigcap_{i=1}^m F_i \neq \emptyset$.

If $k_i > s - r$ for every $1 \leq i \leq m$, then $|\mathcal{F}| \leq \sum_{i=s-r}^{s} \binom{n-1}{i}$.