Exercises Live

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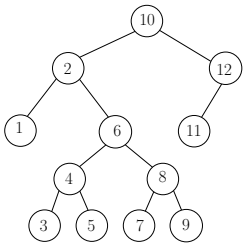
Exercise 1

Consider a random search tree for the elements $\{1, 2, \ldots, n\}$. Let X_n be the random variable counting the number of nodes whose key is smaller than the key of its parent. For example, in the tree below, $X_n = 5$, because the elements $1, 2, 3, 4, 7$ are smaller than their respective parents.



- Draw two search trees on n elements, one where X_n is 0, and one where it is $n - 1$.
- Let Y_n be the number of nodes whose key is *larger* than the key of its parent. What is $X_n + Y_n$?
- Show that $\mathbf{E}[X_n] = \mathbf{E}[Y_n]$.
- Compute $\mathbf{E}[X_n]$. Do this without computing...

Exercise 2

We consider a random search tree for the elements $\{1, 2, \ldots, n\}$. For every $i \in \{1, 2, \ldots, n\}$ compute the expected number of elements in the right subtree of i.

Exercise 3

We toss a coin n times. Let T_n denote the number of times we see two heads in a row. E.g. in the following sequence $HHHTHHT$ the number of times we see two heads in a row is 3. Compute $E[T_n]$.