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Algorithms, Probability, and Computing *Fall 09*
Exercise Set 6

Exercise 1

Which of the following statements are true? Justify your answer by a counterexample, proof, reference to a theorem, etc.

(a) Every network has an integral flow.

(b) Every network has a maximum flow that is integral.

(c) Every network has a maximum flow.

(d) Every network with integral capacities has a unique maximum flow which, moreover, is integral.

(e) A flow in a network is maximum if and only if its value is equal to the capacity of some s-t-cut.

Exercise 2

The set of all flows on a given network is convex. Think this over, formulate it as a precise mathematical statement, and prove it.

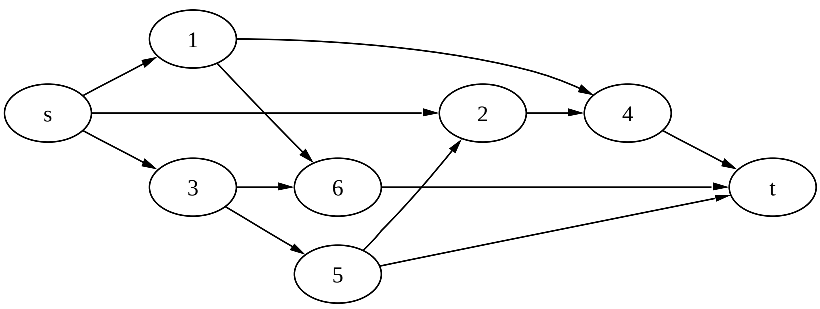
Exercise 3

Show: For every network with m edges there is a sequence of at most m augmentations which produces a maximum flow (beginning with the empty flow).

Hint: Show how to determine m suitable s-t-paths based on a maximum flow.

Exercise 4

Consider the following network where all edge-capacities are 1.



The shortest augmenting path variant of the Ford-Fulkerson algorithm does not take any augmenting path, but chooses a shortest possible one. Simulate this variant on the network shown above. In particular write down the residual network and the current flow in every step. Start with the path $s \to 1 \to 4 \to t$. Why is the resulting flow a maximum flow?

Due-date: 11/02/2009