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Algorithms, Probability, and Computing *Fall 09*
Exercise Set 8

Exercise 1

We consider the following algorithm BetterMinCut for minimum cut, with an input graph G on n vertices and with additional parameters t and T:

```
BetterMinCut(G, t, T):
  H ← RandomContract(G, t)
  return minimum of
      T runs of BasicMinCut(H)
```

```
RandomContract(G, t):
  while |V(G)| > t do
      for random e ∈ E(G)
          G ← G/e
  end while
  return G
```

```
BasicMinCut(G):
  while G has more than 2 vertices do
      pick a random edge e in G
      G ← G/e
  end while
  return the size of the only cut in G
```

In the following, you are asked to analyze the behavior of this algorithm for various settings of t and T. The goal is to make really (!) rough calculations to realize what is going on. You need not worry about integer parts, say. For the probability of success of RandomContract(G, t) use the bound derived in class, which is roughly t^2/n^2, $t \geq 2$.

(a) Consider BetterMinCut called with $t = n/2$ and $T = 2$. How do the probability of success and the running time change compared to the basic guessing algorithm BasicMinCut(G)?

(b) Consider BetterMinCut called with $t = \sqrt{n}$ and $T = n$. Estimate the probability of success. How many times do we need to repeat this algorithm, in order to make the probability of success at least $\frac{1}{2}$? What is the total running time of these repetitions?

(c) Consider now $t = n^\alpha$, $T = n^\beta$, where $\alpha \in (0, 1)$ and $\beta > 0$ are constants. Again estimate the success probability and number N of repetitions needed to make the success probability at least $\frac{1}{2}$. What choice of α and β give the best total running time? (If you cannot determine the very best ones, at least give the best ones you can find.)

Exercise 2

Prove that no graph on n vertices has more than $\binom{n}{2}$ minimum cuts.

Hint: Compare $\binom{n}{2}$ with $P_0(n)$, the success probability of BasicMinCut.

Exercise 3

Suppose we are running the checking algorithm for matrices over GF(2), i.e. numbers are $\{0, 1\}$ with addition and multiplication mod 2. Show that in one iteration the success probability of detecting an error in the supposed product matrix C is exactly $\frac{1}{2}$, in case matrix C is wrong in exactly one row.