

Algorithms, Probability, and Computing *Fall 09*

Exercise Set 10

Exercise 1

Suppose that we have an algorithm for testing the existence of a perfect matching in a given graph, with running time at most $T(n)$ for any n -vertex graph.

- Explain how repeated calls to the algorithm can be used to find a perfect matching if one exists. Estimate the running time of the resulting algorithm.
- How can the algorithm be used for finding a maximum matching in a given graph?

Exercise 2

Let A be a $n \times n$ matrix with 0/1-entries. For $1 \leq i, j \leq n$ let $\epsilon_{i,j}$ be independent random variables, $\epsilon_{i,j} \in_{u.a.r.} \{-1, +1\}$. Let B be the random matrix with $b_{i,j} = \epsilon_{i,j} \cdot a_{i,j}$. In other words, to get B from A we randomly assign signs to the entries of A .

- Show that $\mathbf{E}[\det B] = 0$.
- Show that $\mathbf{E}[(\det B)^2] = \text{per}(A)$. (Challenge)

Exercise 3

Here we use the notation introduced in Definition 6.4 on page 11 of Chapter 6 of the lecture notes (not Prof. Holenstein's chapter). Don't worry, it's only new notation, it is not much different from the concepts introduced in the chapter that was handed out.

Prove the following statements:

- $\text{PCP}(0, \text{poly}(n)) = \text{NP}$,
- $\text{PCP}(O(\log(n)), \text{poly}(n)) = \text{NP}$, and
- $\text{PCP}(O(1), O(1)) = \text{P}$.
- $\text{PCP}(\log \log(n), O(1)) = \text{P}$. Why did we write $\log \log(n)$ and not $O(\log \log(n))$ at this point?