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Algorithms, Probability, and Computing *Fall 09*
Exercises for the First Week

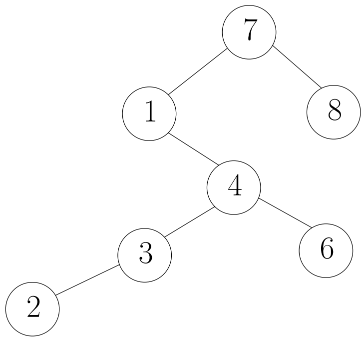
Consider a random search tree for n keys.

Exercise 1

Recall that the expected depth of the smallest key is is $H_n - 1$. Compute the expected depth of the *second smallest* element.

Exercise 2

Let S_n denote the number of keys that are descendants of the smallest key. For example, in the tree below, $S_n = 5$, because the elements $1, 2, 3, 4, 6$ are descendants of 1. Compute $E[S_n]$.



Exercise 3

(Exercise 1.5 from lecture notes). Let $S \subseteq \mathbb{R}$ be a finite set. Show that if the elements of S are inserted in an empty binary search tree in random order—u.a.r. from all permutations of S—then the resulting distribution on $\mathcal{B}_S$ is the same as for random search tress as we defined it.

Exercise 4

(Exercise 1.6 from lecture notes). $n \in \mathbb{N}, n \geq 3$. We choose a random triple ABC of numbers in $\{1, \ldots, n\}$ as follows. First choose $B \in_{u.a.r.} \{2, \ldots, n-1\}$, then $A \in_{u.a.r} \{1, \ldots, B-1\}$ and $C \in_{u.a.r.} \{B+1, \ldots, n\}$. (That is, we have $1 \leq A < B < C \leq n$.)

(1) Given integers $1 \leq a < b < c \leq n$, what is the probability $\Pr[ABC = abc]$?

(2) Is $\{A, B, C\}$ uniformly distributed in $\binom{\{1,\ldots,n\}}{3}$?

(3) Determine $\mathbf{E}[A]$, $\mathbf{E}[B]$ and $\mathbf{E}[C]$.