Exercise 1

Let \( P \subseteq [0,1]^d \) be a finite point set and \( \{\{P_u, P_v\}, u, v\} \) an \( \epsilon^{-1}\)-WSPD of \( P \) with \( 0 < \epsilon < 1 \) computed with the compressed quadtree algorithm. Show that the graph \( G_\epsilon = (P, \{\text{rep}(u), \text{rep}(v) \mid \{P_u, P_v\} \in \epsilon^{-1}\text{-WSPD}\}) \) is connected.

Exercise 2

Let \( G_\epsilon \) be the graph from Exercise 1, where every edge is weighted with the Euclidean distance of its two endpoints. Show that the following function on \( P \times P \) is a metric on \( P \),

\[
d_{G_\epsilon}(p, q) = \text{length of a shortest path in } G_\epsilon \text{ connecting } p \text{ and } q,
\]

where the path length is defined as the sum of the edge lengths in the path.

Exercise 3

Given a finite point set \( P \subseteq [0,1]^d \) with \( |P| = n \). Give an algorithm that computes for every point \( p \in P \) a \((1 + \epsilon)\)-approximate nearest neighbor with \( 0 < \epsilon < 1 \) in time \( O(n \log n + \epsilon^{-d}n) \).

Note that a \((1 + \epsilon)\)-approximate nearest neighbor is at distance at most \((1 + \epsilon)\) times the distance to a nearest neighbor.