Exercise Set 10

Course Webpage: http://www.ti.inf.ethz.ch/ew/courses/ApproxGeom06/

Due date: June 13, 2006

Exercise 1

Let $P \subseteq \mathbb{R}^d$, $|P| = d + 1$ be an affinely independent point set (see Exercise Set 2, Exercise 1). Prove that there exists a unique ball $B_d(c, \rho)$ with all points of $P$ on its boundary, meaning that

$$\|p - c\| = \rho, \quad \forall p \in P.$$

Exercise 2

Prove that for any finite point set $P$, the diameter of the smallest enclosing ball $B(P)$ is at most the diameter of the smallest enclosing axes-parallel box $Q(P)$.

Exercise 3

Let $P \subseteq \mathbb{R}^d$, $|P| = n$. Prove the following statements about smallest enclosing balls and boxes:

(i) There exists $T \subseteq P, |T| \leq 2d$ such that

$$Q(P) = Q(T).$$

(ii) There exists $T \subseteq P, |T| \leq d + 1$ such that

$$B(P) = B(T).$$

Hint: For (ii) use Helly’s Theorem with balls of radius $R_P - \varepsilon$, and $\varepsilon > 0$, centered at the points in $P$. 