Exercise Set 12

Exercise 1

Given a $d \times d$ upper-triangular matrix $A$, with $a_{i,i} = 1$ for $1 \leq i \leq d$, and $|a_{i,j}| \leq 1$ for $1 \leq i < j \leq d$. Let $B = A^{-1}$ be the inverse matrix of $A$. Prove that for $i \in \mathbb{N}$, with $1 \leq i \leq d$, we have $|b_{i-k,i}| \leq 2^{k-1}$, where $k \geq 1$. Also show that this estimate is tight.

Furthermore, show how this implies the following bound on the sum of the absolute values in the $k$-th column of $B$

$$\sum_{i=1}^{k} |b_{i,k}| \leq 2^{k-1}, \quad \text{for } 1 \leq k \leq d,$$

and that all off-diagonal entries of $B$ are bounded by $2^{d-2}$ in absolute value.

**Hint:** $A^{-1}$ is again upper-triangular with all diagonal elements equal to 1.

Exercise 2

Given $P = \{0,1\}^d$. Compute for a given $v \in S^{d-1}$ the “directional width” of $P$ in direction $v$. Try to find a good $\varepsilon$-core-set for this special point set $P$.

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1 we used this bound in the lecture for estimating the directional width of the enclosing cuboid