

Approximate Methods in Geometry**SoSe 2006****Exercise Set 12**Course Webpage: <http://www.ti.inf.ethz.ch/ew/courses/ApproxGeom06/>

Due date: June 27, 2006

Exercise 1

Given a $d \times d$ upper-triangular matrix A , with $a_{i,i} = 1$ for $1 \leq i \leq d$, and $|a_{i,j}| \leq 1$ for $1 \leq i < j \leq d$. Let $B = A^{-1}$ be the inverse matrix of A . Prove that for $i \in \mathbb{N}$, with $1 \leq i \leq d$, we have $|b_{i-k,i}| \leq 2^{k-1}$, where $k \geq 1$. Also show that this estimate is tight.

Furthermore, show how this implies the following bound¹ on the sum of the absolute values in the k -th column of B

$$\sum_{i=1}^k |b_{i,k}| \leq 2^{k-1}, \quad \text{for } 1 \leq k \leq d,$$

and that all off-diagonal entries of B are bounded by 2^{d-2} in absolute value.

Hint: A^{-1} is again upper-triangular with all diagonal elements equal to 1.

Exercise 2

Given $P = \{0,1\}^d$. Compute for a given $v \in S^{d-1}$ the “directional width” of P in direction v . Try to find a good ε -core-set for this special point set P .

¹we used this bound in the lecture for estimating the directional width of the enclosing cuboid