Consider the range space \((X, \mathcal{R})\) where \(X\) is the set of closed axis-parallel squares in the plane, and for every point \(p \in \mathbb{R}^2\) we have a range \(r_p := \{s \in X \mid p \in s\}\) (all squares that contain \(p\)); i.e. we have \(\mathcal{R} := \{r_p \mid p \in \mathbb{R}^2\}\).

Show that this range space has finite VC-dimension. Provide a concrete upper bound on the VC-dimension (you need not determine the VC-dimension exactly).

**Hint:** Observe that a set of squares is shattered if for every subset there is a point in exactly those squares from the subset.

**Exercise 2**

Consider permutations of the first \(n := 6k\) natural numbers \((k\) integral). What is the probability that in a random permutation (uniformly at random from all permutations) all multiples of 3 appear in the first half of the permutation?

**Exercise 3**

Show that \((X, \mathcal{R})\) and \((X, \{X \setminus r \mid r \in \mathcal{R}\})\) have the same VC-dimension.

**Exercise 4**

Let \((X, \mathcal{R})\) and \((X, \mathcal{Q})\) be two range spaces of finite VC-dimension on the same point set \(X\). Show that \((X, \mathcal{R} \cup \mathcal{Q})\) has finite VC-dimension as well, where \(\mathcal{R} \cup \mathcal{Q} := \{r \cup q \mid r \in \mathcal{R}, q \in \mathcal{Q}\}\).

**Hint:** Note that a projection \(\mathcal{R} \cup \mathcal{Q}|_A\) can be obtained by taking unions of ranges in the projections \(\mathcal{R}|_A\) and \(\mathcal{Q}|_A\) (if you like to put this in fancy terms: the operators "projection" and \(\cup\) commute). This allows to bound \(|\mathcal{R} \cup \mathcal{Q}|_A|\) via \(|\mathcal{R}|_A|\) and \(|\mathcal{Q}|_A|\).