Eidgenössische Technische Hochschule Zürich
Ecole polytechnique fédérale de Zurich
Politecnico federale di Zurigo
Swiss Federal Institute of Technology Zurich

Institut für Theoretische Informatik
Lecturers: Bernd Gärtner, Uli Wagner, Emo Welzl
Assistant: Marek Sulovský

May 29, 2007

Approximate Methods in Geometry *Spring 2007*

Exercise Set 10

Course Webpage: `http://www.ti.inf.ethz.ch/ew/courses/ApproxGeom07/`

Due date: June 5, 2007

Exercise 1

Let $P \subseteq \mathbb{R}^d, |P| = n$. Prove the following statements about smallest enclosing balls and boxes:

(i) There exists $T \subseteq P, |T| \leq 2d$ such that

$$Q(P) = Q(T)\,.$$

(ii) There exists $T \subseteq P, |T| \leq d+1$ such that

$$B(P) = B(T)\,.$$

Hint: For (ii) use Helly's Theorem with balls of radius $R_P - \varepsilon$, and $\varepsilon > 0$, centered at the points in P.

Exercise 2

Prove that given a finite point set $P \subset \mathbb{R}^d$ and its optimal enclosing ball $B(P)$ of radius R_P centered in c_P, the following holds:

Lemma. *For any $c \in \mathbb{R}^d$, there exists a point $p \in P$ such that*

1. $||p - c_P|| = R_P$ *and*
2. $||p - c||^2 \geq ||c - c_P||^2 + ||p - c_P||^2$,

Exercise 3

Show, that every ε–core set of smallest enclosing balls has size at least $\Omega(\frac{1}{\varepsilon})$.