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Approximate Methods in Geometry *Spring 2007*

Exercise Set 11

Course Webpage: `http://www.ti.inf.ethz.ch/ew/courses/ApproxGeom07/`

Due date: June 12, 2007

Exercise 1

Show that in each dimension d there is a point set P_d with $\mathrm{Vol}(C(P_d)) \geq e^{\Omega(d \log d)} \mathrm{Vol}(\mathrm{conv}(P_d))$. In other words, show that there are point sets, for which any bounding cuboid has superexponentially (in d) bigger volume than their convex hull.

Exercise 2

In the lecture you have seen that a smallest enclosing cuboid of a point set $P \subseteq \mathbb{R}^d$ approximates the volume of the convex hull of P up to a factor that only depends on d (but not on $|P|$). Prove or disprove whether the same statement holds for a smallest enclosing simplex.

Exercise 3

Given a $d \times d$ upper-triangular matrix A, with $a_{i,i} = 1$ for $1 \leq i \leq d$, and $|a_{i,j}| \leq 1$ for $1 \leq i < j \leq d$. Let $B = A^{-1}$ be the inverse matrix of A. Prove that for $i \in \mathbb{N}$, with $1 \leq i \leq d$, we have $|b_{i-k,i}| \leq 2^{k-1}$, where $k \geq 1$. Also show that this estimate is tight.

Furthermore, show how this implies the following bound[1] on the sum of the absolute values in the k-th column of B

$$\sum_{i=1}^{k} |b_{i,k}| \leq 2^{k-1}, \quad \text{for } 1 \leq k \leq d,$$

and that all off-diagonal entries of B are bounded by 2^{d-2} in absolute value.

Hint: A^{-1} is again upper-triangular with all diagonal elements equal to 1.

[1] we used this bound in the lecture for estimating the directional width of the enclosing cuboid