

Approximate Methods in Geometry **Spring 2007****Exercise Set 11**Course Webpage: <http://www.ti.inf.ethz.ch/ew/courses/ApproxGeom07/>

Due date: June 12, 2007

Exercise 1

Show that in each dimension d there is a point set P_d with $\text{Vol}(C(P_d)) \geq e^{\Omega(d \log d)} \text{Vol}(\text{conv}(P_d))$. In other words, show that there are point sets, for which any bounding cuboid has superexponentially (in d) bigger volume than their convex hull.

Exercise 2

In the lecture you have seen that a smallest enclosing cuboid of a point set $P \subseteq \mathbb{R}^d$ approximates the volume of the convex hull of P up to a factor that only depends on d (but not on $|P|$). Prove or disprove whether the same statement holds for a smallest enclosing simplex.

Exercise 3

Given a $d \times d$ upper-triangular matrix A , with $a_{i,i} = 1$ for $1 \leq i \leq d$, and $|a_{i,j}| \leq 1$ for $1 \leq i < j \leq d$. Let $B = A^{-1}$ be the inverse matrix of A . Prove that for $i \in \mathbb{N}$, with $1 \leq i \leq d$, we have $|b_{i-k,i}| \leq 2^{k-1}$, where $k \geq 1$. Also show that this estimate is tight.

Furthermore, show how this implies the following bound¹ on the sum of the absolute values in the k -th column of B

$$\sum_{i=1}^k |b_{i,k}| \leq 2^{k-1}, \quad \text{for } 1 \leq k \leq d,$$

and that all off-diagonal entries of B are bounded by 2^{d-2} in absolute value.

Hint: A^{-1} is again upper-triangular with all diagonal elements equal to 1.

¹we used this bound in the lecture for estimating the directional width of the enclosing cuboid