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Approximate Methods in Geometry *Spring 2007*

Exercise Set 8

Course Webpage: `http://www.ti.inf.ethz.ch/ew/courses/ApproxGeom07/`

Due date: May 22, 2007

Exercise 1

Show that $(X, \mathcal{R})$ and $(X, \{X \setminus r \mid r \in \mathcal{R}\})$ have the same VC-dimension.

Exercise 2

Let $(X, \mathcal{R})$ and $(X, \mathcal{Q})$ be two range spaces of finite VC-dimension on the same point set X. Show that $(X, \mathcal{R}\hat{\cup}\mathcal{Q})$ has finite VC-dimension as well, where $\mathcal{R}\hat{\cup}\mathcal{Q} := \{r \cup q \mid r \in \mathcal{R}, q \in \mathcal{Q}\}$.

Hint: Note that a projection $\mathcal{R}\hat{\cup}\mathcal{Q}|_A$ can be obtained by taking unions of ranges in the projections $\mathcal{R}|_A$ and $\mathcal{Q}|_A$ (in other words, projection and $\hat{\cup}$ commute). This allows to give a bound on $|\mathcal{R}\hat{\cup}\mathcal{Q}|_A$ in terms of $|\mathcal{R}|_A|$ and $|\mathcal{Q}|_A|$.

Exercise 3

Consider the range space $(X, \mathcal{R})$ where X is the set of closed axis-parallel squares in the plane, and for every point $p \in \mathbb{R}^2$ we have the range $r_p := \{s \in X \mid p \in s\}$ (all squares that contain p) where $\mathcal{R} := \{r_p \mid p \in \mathbb{R}^2\}$.

Show that this range space has finite VC-dimension. Provide a concrete upper bound (it is not necessary to determine the VC-dimension exactly).

Hint: Observe that a set of squares is shattered if and only if for every subset there is a point in exactly the squares from the subset.