**Approximate Methods in Geometry**  
**Spring 2007**

**Exercise Set 8**

**Course Webpage:** [http://www.ti.inf.ethz.ch/ew/courses/ApproxGeom07/](http://www.ti.inf.ethz.ch/ew/courses/ApproxGeom07/)

**Due date:** May 22, 2007

**Exercise 1**

Show that \((X, \mathcal{R})\) and \((X, \{X \setminus r \mid r \in \mathcal{R}\})\) have the same VC-dimension.

**Exercise 2**

Let \((X, \mathcal{R})\) and \((X, \mathcal{Q})\) be two range spaces of finite VC-dimension on the same point set \(X\). Show that \((X, \mathcal{R} \cup \mathcal{Q})\) has finite VC-dimension as well, where \(\mathcal{R} \cup \mathcal{Q} := \{r \cup q \mid r \in \mathcal{R}, q \in \mathcal{Q}\}\).

**Hint:** Note that a projection \(\mathcal{R} \cup \mathcal{Q}|_A\) can be obtained by taking unions of ranges in the projections \(\mathcal{R}|_A\) and \(\mathcal{Q}|_A\) (in other words, projection and \(\cup\) commute). This allows to give a bound on \(|\mathcal{R} \cup \mathcal{Q}|_A|\) in terms of \(|\mathcal{R}|_A|\) and \(|\mathcal{Q}|_A|\).

**Exercise 3**

Consider the range space \((X, \mathcal{R})\) where \(X\) is the set of closed axis-parallel squares in the plane, and for every point \(p \in \mathbb{R}^2\) we have the range \(r_p := \{s \in X \mid p \in s\}\) (all squares that contain \(p\)) where \(\mathcal{R} := \{r_p \mid p \in \mathbb{R}^2\}\).

Show that this range space has finite VC-dimension. Provide a concrete upper bound (it is not necessary to determine the VC-dimension exactly).

**Hint:** Observe that a set of squares is shattered if and only if for every subset there is a point in exactly the squares from the subset.