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Approximate Methods in Geometry *Spring 2007*

Exercise Set 9

Course Webpage: `http://www.ti.inf.ethz.ch/ew/courses/ApproxGeom07/`

Due date: May 29, 2007

Exercise 1

For real numbers a, b and c, we define the region

$$p(a,b,c) := \{(x,y) \in \mathbb{R}^2 \mid y^2 \geq a(x-b)^2 + c\}$$

Now consider the range space $(\mathbb{R}^2, \mathcal{P})$ with $\mathcal{P} := \{p(a,b,c) \mid (a,b,c) \in \mathbb{R}^3\}$.

(1) Give three points that are shattered by $\mathcal{P}$ (give their coordinates and specify the ranges used for shattering).
(2) Show that the VC-dimension of the range space is 3 (employ linearization, i.e. use a map similar to the lifting map).

Exercise 2

Let $(X, \mathcal{R})$ be a range space of finite VC-dimension d. Every pair of ranges q and r partitions X into four sets: $X \setminus (q \cup r)$, $q \cap r$, $q \setminus r$, and $r \setminus q$. Now let $\mathcal{R}'$ be the set of ranges obtained by taking for every pair of ranges the four sets as above as ranges into $\mathcal{R}'$. Find a concrete upper bound on the VC-dimension of $(X, \mathcal{R}')$ in terms of d. Try to make the bound as good as you can.

Exercise 3

We are given a set P of n (where $n \geq 5$) points in general position in the plane, i.e. no three points on a common line. Show that there is a positive constant c such that a randomly chosen subset $F \in \binom{P}{4}$ (u.a.r.) is in convex position (i.e. forms a convex quadrilateral) with probability at least c.

Hint: Is there some c_5 valid for all 5 point sets P? Now, what happens if we first pick a random subset $X \in \binom{P}{5}$ and then pick a random $Y \in \binom{X}{4}$? What distribution does Y have? What does it say about c?

Remark: This exercise seems to have nothing to do with ε-nets — and so it is. However the idea of the argument follows the lines of the ε-net proof: First choose a small subset (or sequence), and then take a subset of the subset (or permute and consider some initial segment of the sequence). This is a strong paradigm for probabilistic arguments.