Approximate Methods in Geometry Spring 2008

Exercise Set 7

Course Webpage: http://www.ti.inf.ethz.ch/ew/courses/ApproxGeom08/

Due date: April 15, 2008

Exercise 1
Show that there is a constant $c$ such that for every $\varepsilon$ there is a dimension $d_\varepsilon$ and a finite point set $P_\varepsilon \subseteq \mathbb{R}^{d_\varepsilon}$ with every $\varepsilon$-core set of $P_\varepsilon$ of size at least $c$.

Exercise 2
Show that in each dimension $d$ there is a point set $P_d$ with $\text{Vol}(C(P_d)) \geq e^{\Omega(d \log d)} \text{Vol}(%00ed\text{conv}(P_d))$. In other words, show that there are point sets, for which any bounding cuboid has superexponentially (in $d$) bigger volume than their convex hull.

Exercise 3
Given a $d \times d$ upper-triangular matrix $A$, with $a_{i,i} = 1$ for $1 \leq i \leq d$, and $|a_{i,j}| \leq 1$ for $1 \leq i < j \leq d$. Let $B = A^{-1}$ be the inverse matrix of $A$. Prove that for $i \in \mathbb{N}$, with $1 \leq i \leq d$, we have $|b_{i-k,i}| \leq 2^{k-1}$, where $k \geq 1$. Also show that this estimate is tight.

Furthermore, show how this implies the following bound\footnote{we used this bound in the lecture for estimating the directional width of the enclosing cuboid} on the sum of the absolute values in the $k$-th column of $B$

$$\sum_{i=1}^{k} |b_{i,k}| \leq 2^{k-1}, \quad \text{for } 1 \leq k \leq d,$$

and that all off-diagonal entries of $B$ are bounded by $2^{d-2}$ in absolute value.

Hint: $A^{-1}$ is again upper-triangular with all diagonal elements equal to 1.