Exercise 1
Let \( P := \{0, 1\}^d \). Given a \( v \in S_{d-1} \), calculate the width \( w_v(P) \) of \( P \) in the direction \( v \).

What is the smallest \( \epsilon \)-core set for the directional width of \( P \) (now in all directions at once - as it is defined in the lecture notes) for \( 0 < \epsilon < \frac{1}{d} \)?

Exercise 2
Assume you are given a compact set \( K \subseteq \mathbb{R}^2 \) such that \( w_u(K) = w_v(K) \) for all directions \( u, v \in S_{d-1} \) (i.e. \( K \) has the same width in all directions). Does this automatically imply that \( \text{conv}(K) \) is a disk, or are there other possible shapes?

Exercise 3
The aspect ratio of a compact set \( K \) is defined as
\[
\frac{\max_{v \in S_{d-1}} w_v(K)}{\min_{v \in S_{d-1}} w_v(K)}.
\]

Prove that for any compact set \( K \) in \( \mathbb{R}^d \) for which \( \min_{v \in S_{d-1}} w_v(K) \neq 0 \), there is an affine transformation into set \( K' \) such that the aspect ratio of \( K' \) is bounded by \( d \).