Approximate Methods in Geometry  
Spring 2008

Exercise Set 9

Course Webpage: http://www.ti.inf.ethz.ch/ew/courses/ApproxGeom08/

Due date: April 29, 2008

Exercise 1

An annulus with parameters \((c, r, R)\), \(c \in \mathbb{R}^d, r \leq R\) is the set \(\{x \in \mathbb{R}^d \mid r \leq \|x - c\| \leq R\}\) (this means it is a region between two concentric spheres with center \(c\) and radii \(r\) and \(R\)). Prove the following statement:

For any finite point set \(P \subseteq \mathbb{R}^d\) and \(\epsilon > 0\), there exists \(S \subseteq P\) (a core set, whose size is a function of \(d\) and \(\epsilon\)) such that for any \(c \in \mathbb{R}^d\), the smallest enclosing annulus of \(S\) with center \(c\) is a \((1 - \epsilon)\)-approximation of the smallest enclosing annulus of \(P\) with center \(c\). Here, the approximation is with respect to the measure \(R^2 - r^2\) (i.e. difference of the squared radii of the annulus).

Exercise 2

Give a family of continuous functions \(f_i : \mathbb{R}^d \to \mathbb{R}\) for \(i = 1, \ldots, n\) (for every \(n \geq 2\)) such that for all \(1 > \epsilon > 0\), every \((1 - \epsilon)\) core set of \(\{f_i\}\) with respect to extent must contain all the \(f_i\) (from what we proved, the \(f_i\) cannot be polynomials of some fixed degree).

Exercise 3

Show that the width (the smallest directional width) of a point set \(P \subseteq \mathbb{R}^2, |P| = n\) can be computed in time \(O(n \log n)\).

Hint: There is an \(O(n \log n)\) time algorithm for computing the convex hull of a set of \(n\) points in \(\mathbb{R}^2\).