

Approximate Methods in Geometry **Spring 2008****Exercise Set 12**Course Webpage: <http://www.ti.inf.ethz.ch/ew/courses/ApproxGeom08/>

Due date: May 20, 2008

Exercise 1For $\Phi_d(n) := \sum_{i=0}^d \binom{n}{i}$, $d \in \mathbb{N}_0 \cup \{-1\}$, $n \in \mathbb{N}_0$, show that

$$\Phi_d(n) = \begin{cases} 0 & d = -1, \\ 1 & n = 0 \text{ and } d \geq 0, \text{ and} \\ \Phi_d(n-1) + \Phi_{d-1}(n-1) & \text{otherwise.} \end{cases}$$

Exercise 2What is the VC-dimension of axis parallel rectangles in the plane? (To be precise, of the range space which has the points in the plane \mathbb{R}^2 as points, and closed axis parallel rectangles as ranges.)**Exercise 3**Let $\mathcal{I}^{k\cup}$ be the set of k -fold unions of intervals of \mathbb{R} (i.e. $\mathcal{I}^{k\cup} = \{\bigcup_{i=1}^k r_i \mid r_i \in \mathcal{I} \text{ for all } i\}$).(1) Determine the VC-dimension of $\mathcal{I}^{2\cup}$.(2) Determine the VC-dimension of $\mathcal{I}^{k\cup}$ for $k \in \mathbb{N}$.

(Clearly, an answer to (2) implies a solution to (1), in particular.)

Exercise 4Consider the range space of complements of axis parallel rectangles in the plane. Suppose we are given a set A of n points in the plane.(1) Show that if the complement of an axis parallel rectangle r contains more than εn points from A , then there is a halfplane h bounded by a vertical or a horizontal line, so that h is disjoint from r (i.e. it is contained in the complement of r) and $|h \cap A| > \frac{\varepsilon}{4}n$.(2) Show that, for c an appropriate constant, sampling at least $\frac{c\lambda}{\varepsilon}$ points in A will give an ε -net for complements of rectangles with probability at least $1 - e^{-\lambda}$.(Hint: Use here the fact that it suffices to sample a point in each one of a fixed set of four halfplanes with more than $\frac{\varepsilon}{4}n$ points.)