Computational Geometry Exercise Set 10 WS06/07

Exercise 1 (10 points)

Let \( L \) be a set of \( n \) lines in \( \mathbb{R}^2 \) no three of which pass through a common point. Suppose that all lines from \( P \subseteq L \) are parallel to each other, no two lines from \( L \setminus P \) are parallel to each other, and no line from \( L \setminus P \) is parallel to those from \( P \). Determine the number of vertices, edges, and faces of the arrangement \( A(L) \) in terms of \( n \) and \( k := |P| \).

Exercise 2 (10 points)

For an arrangement \( A \) of a set of \( n \) lines in \( \mathbb{R}^2 \), let \( \mathcal{F} := \bigcup_{C \text{ cell of } A} \overline{C} \) denote the union of the closure of all bounded cells. Show that the complexity (number of vertices and edges) of \( \mathcal{F} \) is \( O(n) \).

Exercise 3 (10 points)

Given a set \( S \) of \( n \) line segments in \( \mathbb{R}^2 \). Describe a data structure for \( S \) to answer in \( O(\log n) \) time how many segments from \( S \) a given query line intersects. Storage should be \( O(n^2) \) and preprocessing time \( O(n^2 \log n) \).

Due date: January 22, 2007