

Computational Geometry**Exercise Set 5****WS06/07**URL: <http://www.ti.inf.ethz.ch/ew/courses/CG06/>**Exercise 1 (10 points)**

Let M be a set of n real numbers. For $1 \leq r \leq n$, choose a set R of r numbers randomly from M , each set in $\binom{M}{r}$ with the same probability. Determine the expected value of the number of elements of M which lie outside the interval $I(R) := [\min R, \max R]$.

Exercise 2 (10 points)

You are given

- a star-shaped polygon $P \subset \mathbb{R}^2$, represented as a doubly connected list of its vertices $V(P)$,
- and a point $c \in P$ (not necessarily in $V(P)$), such that for all $p \in P$ the line segment \overline{cp} is contained in P .

Describe an algorithm which triangulates P in linear time. The algorithm could for example output all edges of the triangulation, that are not already edges of the polygon.

Exercise 3 (10 points)

This exercise is about a variant of the Voronoi Diagram:

Let $P = \{p_1, \dots, p_n\}$ be a set of points in \mathbb{R}^2 . Define $\text{Far}_P(i) := \{x \in \mathbb{R}^2 : \|x - p_i\| \geq \|x - p_j\|, \text{ for all } p_j \in P\}$. The Farthest Point Voronoi Diagram of P (FVD(P)) is the set of all cells $\text{Far}_P(p_i)$ for $i = 1..n$.

1. Show: $\text{Far}_P(p_i)$ is a convex polytope for all $i \in \{1, \dots, n\}$.
2. Show: $\text{Far}_P(p_i)$ is not empty if and only if p_i is an extreme point of P , for all $i \in \{1, \dots, n\}$.
3. Draw the Voronoi Diagram and the Farthest Point Voronoi Diagram of three points. Determine for each region the closest and the farthest point.
4. Like exercise 3.3, but now for four points which are not in convex position.
5. Like exercise 3.3, now for four points in convex position.