Exercise 1  (10 points)

a) Let $S$ be a set of $n$ disjoint line segments in the plane. Determine all segments in $S$ that intersect a ray which starts in a query point $(q_x, q_y)$ and runs vertically upwards. Find a data structure with size $O(n \log n)$ and query time $O(\log n + k)$, where $k$ is the number of intersecting segments.

b) Now we only want to find the first segment that is intersected by the query ray. Find a data structure with expected size $O(n)$ and expected query time $O(\log n)$.

Exercise 2  (10 points)

Show that a Range Tree for $n$ points can be constructed in $O(n \log n)$ time.

Exercise 3  (10 points)

Find a data structure for $n$ points in the plane which allows to answer dominance queries efficiently. For a given query point $p = (x, y)$ find in $O(\log n + k)$ time all $k$ points that dominate $p$. A point $q = (x', y')$ dominates $p$ if $x' > x$ and $y' > y$. 

Due date: January 8, 2007