Eidgenössische Technische Hochschule Zürich
Ecole polytechnique fédérale de Zurich
Politecnico federale di Zurigo
Swiss Federal Institute of Technology Zurich

Institute for Theoretical Computer Science
Dr. B. Gärtner, Dr. M. Hoffmann and Marek Sulovský

1.10.2007

Computational Geometry — Exercise Set 1 — HS07

URL: http://www.ti.inf.ethz.ch/ew/courses/CG07/

Exercises

In every lecture we provide you with an exercise sheet. You should solve it in written form and return the solutions at the beginning of the subsequent lecture. Solving the exercises in teams is not allowed. Your solutions will be graded. If you reach at least 80% of all possible points you will get the grade 6.0, with 40% of all points you will get a 4.0.

Exam

There will be an oral exam of 15 minutes during the examination period. Your final grade consists to 50% of the grade for the exam and to 50% of the grade for the exercises.

Exercise 1 (10 points)

A set $S \subset \mathbb{R}^d$ is *star-shaped* $\iff$ there exists a point $c \in S$, such that for every point $p \in S$ the line segment $\overline{cp}$ is contained in S. A set $S \subset \mathbb{R}^d$ is a *pseudotriangle* $\iff$ it is a simple polygon and has exactly three convex vertices (see Figure 1).

In the following we consider subsets of $\mathbb{R}^d$. Prove or disprove:

a) Every star-shaped set is convex.

b) Every convex set is star-shaped.

c) The intersection of two convex sets is convex.

d) The union of two convex sets is convex.

e) The intersection of two star-shaped sets is star-shaped.

f) The intersection of a convex set with a star-shaped set is star-shaped.

g) Every pseudotriangle is star-shaped.

Exercise 2 (10 points)

Let $P = \{p_1, \ldots, p_n\}$ be a set of $n \geq 3$ points in $\mathbb{R}^2$ and let $q \in \text{conv}(P)$ be another point. Prove that there exist three points p_i, p_j and p_k, $1 \leq i, j, k \leq n$, such that $q \in \text{conv}(\{p_i, p_j, p_k\})$.

see reverse

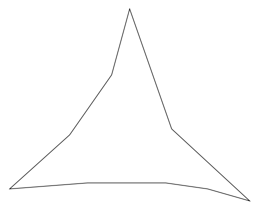

Figure 1: A pseudotriangle

Exercise 3 (10 points)

Consider three points $p, q, r \in \mathbb{R}^2$, given by their Cartesian coordinates $p = (p_x, p_y)$, $q = (q_x, q_y)$ and $r = (r_x, r_y)$. Show: the sign of the determinant

$$\begin{vmatrix} 1 & p_x & p_y \\ 1 & q_x & q_y \\ 1 & r_x & r_y \end{vmatrix}$$

determines if r lies to the right, to the left or on the directed line through p and q.

Exercise 4 (10 points)

Let $P \subset \mathbb{R}^2$ be a convex polygon, given as an array $\mathtt{p[0]} \ldots \mathtt{p[n]}$ of its $n+1$ vertices in counter clockwise order. Describe an algorithm with running time $O(\log(n))$ which determines whether a point q lies inside, outside or on the boundary of P.

Due date: 8.10.2007, 13h15