Eidgenössische Technische Hochschule Zürich
Ecole polytechnique fédérale de Zurich
Politecnico federale di Zurigo
Swiss Federal Institute of Technology Zurich

Institute for Theoretical Computer Science *8.10.2007*
Dr. B. Gärtner, Dr. M. Hoffmann and Marek Sulovský

Computational Geometry Exercise Set 2 HS07

URL: http://www.ti.inf.ethz.ch/ew/courses/CG07/

Exercise 1 (10 points)

We consider the mapping that assigns to a point $p = (p_x, p_y) \in \mathbb{R}^2$ the nonvertical line $p^* : y = p_x x - p_y$ and, vice versa, assigns to the nonvertical line $g : y = mx + b$, $m, b \in \mathbb{R}$ the point $g^* = (m, -b)$.

1. Show that this mapping preserves incidences, i.e. for a point p and a line g it holds $p \in g \iff g^* \in p^*$.
2. Show that this mapping preserves order, i.e. for a point p and a line g it holds: p is above $g \iff g^*$ is above p^*.
3. Describe the image of the following point sets under this mapping
   (a) a half plane
   (b) $k \geq 3$ colinear points
   (c) a line segment
   (d) the boundary points of the upper convex hull of a finite point set.
4. Consider the parabola $\mathcal{P} : y = x^2/2$. For $p \in \mathcal{P}$ characterize p^* with respect to $\mathcal{P}$.

Exercise 2 (10 points)

The *lower envelope* of a set G of non-vertical lines in $\mathbb{R}^2$ is defined to be the set of all points p such that

- p lies on (at least) one line in G and
- there is no line in G, which is strictly below p.

Describe an $O(n \log n)$ algorithm which computes the lower envelope of a set of n non-vertical lines in $\mathbb{R}^2$.

Exercise 3 (10 points)

Consider a set $M \subset \mathbb{R}^2$ of n points. Describe an algorithm which decides in linear time if another point q lies in $\mathrm{conv}(M)$.

See reverse

Exercise 4 (10 points)

For a sequence of n pairwise distinct numbers $y_1, \ldots, y_n$ consider the sequence of pairs $(\min\{y_1, \ldots, y_i\}, \max\{y_1, \ldots, y_i\})_{i=0,1,\ldots,n}$ ($\min \emptyset := +\infty, \max \emptyset := -\infty$). How often do these pairs change in expectation if the sequence is permuted randomly, each permutation appearing with the same probability? Determine the expected value.

Due date: 15.10.2007, 13h15