

Computational Geometry**Exercise Set 8****HS07**URL: <http://www.ti.inf.ethz.ch/ew/courses/CG07/>**Exercise 1 (10 points)**

Let L be a set of n lines in \mathbb{R}^2 no three of which pass through a common point. Suppose that all lines from $P \subseteq L$ are parallel to each other, no two lines from $L \setminus P$ are parallel to each other, and no line from $L \setminus P$ is parallel to those from P . Determine the number of vertices, edges, and faces of the arrangement $\mathcal{A}(L)$ in terms of n and $k := |P|$.

Exercise 2 (10 points)

For an arrangement \mathcal{A} of a set of n lines in \mathbb{R}^2 , let $\mathcal{F} := \bigcup_{C \text{ is cell of } \mathcal{A}} \overline{C}$ denote the union of the closure of all bounded cells. Show that the complexity (number of vertices and edges of the arrangement lying on the boundary) of \mathcal{F} is $O(n)$.

Exercise 3 (10 points)

Given a set of lines in the plane with no three intersecting in a common point, form a graph G whose vertices are the intersections of the lines, with two vertices adjacent if they appear consecutively on one of the lines. Prove that $\chi(G) \leq 3$. (χ is the chromatic number of the graph)

Exercise 4 (10 points)

Given a set S of n line segments in \mathbb{R}^2 . Describe a data structure for S to answer in $O(\log n)$ time how many segments from S a given query line intersects. Storage should be $O(n^2)$ and preprocessing time $O(n^2 \log n)$.