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Computational Geometry — Exercise Set 9 — HS07

URL: http://www.ti.inf.ethz.ch/ew/courses/CG07/

Exercise 1 (10 points)

Consider the following recursion $T(n, d)$ for $n, d \in \mathbb{N}_0$.

$$\begin{aligned} T(0, d) &= 0\,, \\ T(n, 0) &= 0\,, \\ T(n, d) &\leq 1 + T(n-1, d) + \frac{d}{n} T(n-1, d-1)\,, \text{ for } n, d > 0. \end{aligned}$$

Show inductively that $T(n, d) \leq c_d n$ where $c_d = d! \sum_{i=0}^{d} \frac{1}{i!}$

One can also show (but you are not asked to do so) that $c_d = d! \sum_{i=0}^{d} \frac{1}{i!} = \lfloor e \cdot d! \rfloor$.

This recursion gives an estimate for the expected number of insphere-tests of the randomized algorithm for the computation of the smallest enclosing ball of a point set in $\mathbb{R}^{d-1}$.

Exercise 2 (10 points)

Find a set $P \subset \mathbb{R}^2$ of n points for which the randomized algorithm from the lecture needs $\Omega(n^2)$ incircle tests in the worst case in order to compute the smallest enclosing ball of P.

Exercise 3 (10 points)

Prove that any recursive call to MINIBALL(P,R) triggered by MINIBALL(S, $\emptyset$) has an affinely independent point set R.

Remark: A finite set P of points is called affinely independent, if there exists no nontrivial combination $\sum_{p \in P} \alpha_p p = 0$ with $\sum_{p \in P} \alpha_p = 0$. By nontrivial we mean that not all α_p are 0.

Due date: 3.12.2007