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Computational Geometry Exercise Set 10 HS07

URL: http://www.ti.inf.ethz.ch/ew/courses/CG07/

Exercise 1 (10 points)

The k–sum problem is defined as follows: given k sets $S_1, \ldots, S_k$ of n integers each, are there $a_1 \in S_1, \ldots, a_k \in S_k$ such that $\sum_{i=1}^{k} a_i = 0$? Find an algorithm, which solves k–sum in $O(n^{\lceil \frac{k}{2} \rceil} \log n)$ for k even and in $O(n^{\lceil \frac{k}{2} \rceil})$ for k odd.

Note that k is not part of the input and therefore is considered constant.

Exercise 2 (10 points)

An important source of geometric problems in mobile applications involves objects moving in time.

Consider the following setting: there are n cars driving with constant velocities on a straight highway. We can model them as n moving points on a line such that point p_i is at time t at position $s_i + v_i t$. Above these cars flies a police helicopter. It has a camera pointed straight down with a view angle $0 < \theta < \pi$ and needs to see all the cars. Since the pilot is afraid of heights (well, bad career choice) he wants to keep flying as low as possible all the time.

(a) given a time interval $T = [t_0, t_1]$ compute a function $h(t)$ on T which gives the position of the helicopter in time, such that its y coordinate is the lowest possible and the camera sees all cars at every moment. Note: By compute, we ask you to give a fast algorithm, which describes the whole function (not only to describe a single function value).

(b) Suppose there is a speed limit v_{max} on the highway, which means $\forall i : |v_i| \leq v_{max}$. Express the maximal velocity this helicopter might need to fly as a function of θ and v_{max}

Exercise 3 (10 points)

Consider an arrangement of n lines in the plane in general position. A edge of the arrangement is on the k–level $\mathcal{L}_k$ if it has exactly k lines of the arrangement strictly below it. Objective of this exercise is to establish an $O(n\sqrt{k})$ upper bound on the combinatorial complexity (in this case number of edges) of $\mathcal{L}_k$. We classify vertices of the k–level (endpoints of edges in $\mathcal{L}_k$) into two groups: valley vertices (where the level moves from a line with a lower slope to a line with a higher slope) and peak vertices (from higher slope to lower slope).

Let l_i denote the line with the i-th lowest slope. For any edge e of the arrangement define a potential function $\psi(e) = \sum_{l_i : e \text{ lies on or above } l_i} i$.

(a) Let $(e_1, \ldots, e_m)$ be the sequence of all edges of $\mathcal{L}_k$ from left to right and let $\psi_i := \psi(e_i)$. Show that if e_i and e_{i+1} form a peak vertex, then $\psi_i > \psi_{i+1}$ and for valley vertices $\psi_i = \psi_{i+1}$.

(b) Prove that $\psi_m - \psi_1 \in O(nk)$

(c) Define $\delta_i := \psi_{i+1} - \psi_i$. We classify these potential changes into large ($\delta_i > \sqrt{k}$) and small ($0 < \delta_i \leq \sqrt{k}$). Prove that total number of large potential changes is $O(n\sqrt{k})$

(d) Prove that total number of small potential changes is $O(n\sqrt{k})$

(e) Combine the previous to obtain an $O(n\sqrt{k})$ upper bound on the number of peak vertices. Conclude an $O(n\sqrt{k})$ upper bound on the number of valley vertices, thus obtaining the same bound on the number of edges in $\mathcal{L}_k$.

Due date: 10.12.2007