Institute for Theoretical Computer Science *10.12.2007*
Dr. B. Gärtner, Dr. M. Hoffmann and Marek Sulovský

Computational Geometry   Exercise Set 11   HS07

URL: http://www.ti.inf.ethz.ch/ew/courses/CG07/

This exercise serie is not obligatory and might not be discussed in the exercise session, though you might still want to submit it because you can earn precious bonus point for these exercises.

Exercise 1 (10 points)

The 3–sum′ problem is defined as follows: given 3 sets S_1, S_2, S_3 of n integers each, are there $a_1 \in S_1, a_2 \in S_2, a_3 \in S_3$ such that $a_1 + a_2 + a_3 = 0$?

Prove that this 3–sum′ and 3–sum as defined on the lecture (there the sets were the same) are equivalent. By equivalence we mean that 3-sum can be reduced to 3-sum′ and vice-versa, 3–sum′ can be reduced to 3–sum.
By reduction we mean, that any instance I of size n of one can be transformed in subquadratic time into an instance I' of size $O(n)$ of the other, such that I is has a solution if and only if I' has a solution.

Note: Such an equivalence implies that if there was a subquadratic algorithm for 3–sum then there would be a subquadratic algorithm for 3–sum′ and vice-versa.

Exercise 2 (10 points)

Prove that $\lambda_2(n) = 2n - 1$.

Exercise 3 (10 points)

Prove the following statements about pseudotriangulations:

- Let (R, P) be a pointgon on $|P| = n$ points and r reflex vertices in the polygon R. Let T be a pseudotriangulation of (R, P) with n_X non–pointed vertices. Then T has $2n - 3 + (n_X - r)$ edges and $n - 2 + (n_x - r)$ pseudo–triangles.

- Let T be a graph embedded on a set P of n points. The following are equivalent:
  1. T is a pseudotriangulation of P with the minimum possible number of vertices
  2. T is a pointed pseudotriangulation of P
  3. T is a pseudotriangulation of P with $m = 2n - 3$ edges

Note: *Pointgon* (R, P) is a polygon R and a specified set of points P containing all vertices of R and maybe some other points in its interior.
A vertex of a plane graph is called *pointed* if some pair of consecutive edges spans a reflex angle. It is called *non-pointed* otherwise.

Due date: 17.12.2007