Exercise 1 (10 points)

A set $P$ of $n$ points in the plane is said to be in $\varepsilon$-general position for $\varepsilon > 0$ if no three points of the form

$$p + (x_1, y_1), q + (x_2, y_2), r + (x_3, y_3)$$

are collinear, where $p, q, r \in P$ and $|x_i|, |y_i| < \varepsilon$ for $i = 1, 2, 3$. In words: $P$ remains in general position under changing point coordinates by less than $\varepsilon$ each.

Give an algorithm with runtime $O(n^2)$ for checking whether a given point set $P$ is in $\varepsilon$-general position.

Exercise 2 (10 points)

a) You are given a set of $n$ pairwise disjoint line segments. Find an algorithm to answer vertical ray shooting queries in $O(\log n)$ time. That is, preprocess the data such that given a query point $q$ you can report in $O(\log n)$ time which segment is the first above $q$ (or if there are none). Analyze the running time and the space consumption of the preprocessing.

b) What happens if we allow intersections of the line segments? Explain in a few words how you have to adapt your solution and how the time and space complexity would change.

Exercise 3 (30 points)

Perform a small research as you did in Homework 1. Choose one of the following problems/topics to investigate.

a) Additively weighted Voronoi diagrams

b) Higher order Delaunay triangulations

c) Halfplane range searching

d) Ham-sandwich cuts

e) Delaunay refinement meshing

Your report should contain

- an informal description of the problem(s) using your own words,
- a precise formal definition of the problem(s),
- a chronological list of the important results,
- the current state of the problem(s), in particular, open questions and possible future directions,
- a complete list of references.