Exercise 1

A set $S \subseteq \mathbb{R}^d$ is star-shaped $\iff$ there exists a point $c \in S$, such that for every point $p \in S$ the line segment $cp$ is contained in $S$. A set $S \subseteq \mathbb{R}^d$ is a pseudotriangle $\iff$ it is a simple polygon and has exactly three convex vertices (see Figure 1).

In the following we consider subsets of $\mathbb{R}^d$. Prove or disprove:

a) Every star-shaped set is convex.

b) Every convex set is star-shaped.

c) The intersection of two convex sets is convex.

d) The union of two convex sets is convex.

e) The intersection of two star-shaped sets is star-shaped.

f) The intersection of a convex set with a star-shaped set is star-shaped.

g) Every pseudotriangle is star-shaped.

Exercise 2

Consider three points $p, q, r \in \mathbb{R}^2$, given by their Cartesian coordinates $p = (p_x, p_y)$, $q = (q_x, q_y)$ and $r = (r_x, r_y)$. Show: the sign of the determinant

\[
\begin{vmatrix}
1 & p_x & p_y \\
1 & q_x & q_y \\
1 & r_x & r_y \\
\end{vmatrix}
\]

determines if $r$ lies to the right, to the left or on the directed line through $p$ and $q$.

Figure 1: A pseudotriangle
Exercise 3

Let $P \subseteq \mathbb{R}^2$ be a convex polygon, given as an array $p[0] \ldots p[n]$ of its $n+1$ vertices in counter clockwise order.

(a) Describe an algorithm with running time $O(\log(n))$, which determines whether a point $q$ lies inside, outside or on the boundary of $P$.

(b) Describe an algorithm with running time $O(\log(n))$, which finds a (right) tangent to $P$ from a query point $q$ outside $P$ (i.e. you should find a vertex $p[i]$, s.t. whole $P$ is contained in a (left) halfplane determined by the line $qp[i]$).

*Exercise 4  (Caratheodory's Theorem)

Let $P = \{p_1, \ldots, p_n\}$ be a set of $n \geq d+1$ points in $\mathbb{R}^d$ and let $q \in \text{conv}(P)$ be another point. Prove that there exists a subset $P' \subseteq P$ consisting of $d+1$ points such that $q \in \text{conv}(P')$.

*Exercise 5

Prove or disprove: The convex hull of a closed subset of $\mathbb{R}^d$ is closed.