Exercise 1

Suppose that you have an algorithm $A$ for solving feasible linear programs of the form

\[(LP) \quad \text{maximize} \quad c^T x \\
\text{subject to} \quad Ax \leq b,\]

where feasible means that there exists $\bar{x} \in \mathbb{R}^d$ such that $A\bar{x} \leq b$. Extend algorithm $A$ such that it can deal with arbitrary (not necessarily feasible) linear programs of the above form.

Exercise 2

Prove that all sets $R$ of constraints that arise during a call to algorithm $\mathcal{CP}(H,\emptyset)$ are independent, meaning that the set

$$\{x \in \mathbb{R}^d : a_h x = b_h, h \in R\}$$

of points that satisfy all constraints in $R$ with equality has dimension $d - |R|$.

Exercise 3

In order to adapt Seidel’s randomized linear programming algorithm to the problem of computing smallest enclosing balls, we need the following statements.

(i) Let $P, R \subseteq \mathbb{R}^d, P \cap R = \emptyset$. If there exists a ball that contains $P$ and has $R$ on the boundary, then there is also a unique smallest such ball which we denote by $B(P, R)$.

(ii) Let $P, R \subseteq \mathbb{R}^d, P \cap R = \emptyset$. If $B(P, R)$ exists and $p \in P$ satisfies $p \not\in B(P \setminus \{p\}, R)$, then $p$ is on the boundary of $B(P, R)$, meaning that $B(P, R) = B(P \setminus \{p\}, R \cup \{p\})$.

Prove these two statements!