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# *Discrete Geometry* — *Problem Set 8*

**Course Webpage:** `http://www.ti.inf.ethz.ch/ew/courses/DG07/`

**Due date:** June 14, 2007

## Exercise 8.1

([3]+[2] points)

Use the Szemerédi-Trotter theorem to show that $n$ points in the plane determine at most

(a) $O(n^{7/3})$ triangles of unit area,

(b) $O(n^{7/3})$ triangles with a fixed given angle $\alpha$.

## Exercise 8.2

([2]+[3] points)

(a) Using the Szemerédi-Trotter theorem, show that the maximum possible number of distinct lines such that each of them contains at least $k \geq 2$ points of a given $m$-point set $P$ in the planes is $O(m^2/k^3 + m/k)$.

(b) Prove that such lines have at most $O(m^2/k^2 + m)$ incidences with $P$.

## Exercise 8.3

([4] points)

Show that an $n$-point set in $\mathbb{R}^4$ may determine $\Omega(n^2)$ unit distances.

## Exercise 8.4

([2] points)

Prove that for all $m, n$ with $n^2 \geq m$ and $m^2 \geq n$, we have

$$I(m, n) = \Omega(n^{2/3}m^{2/3}).$$

## Exercise 8.5

([3] points)

In a manner similar to the proof for point–line incidences covered in class, prove the bound

$$I_{1\text{circ}}(n, n) = O(n^{4/3}),$$

where $I_{1\text{circ}}(m, n)$ denotes the maximum possible number of incidences between $m$ points and $n$ unit circles in the plane (be careful in handling possible multiple edges in the considered topological graph!).