Eidgenössische Technische Hochschule Zürich
Ecole polytechnique fédérale de Zurich
Politecnico federale di Zurigo
Swiss Federal Institute of Technology Zurich

Institute for Theoretical Computer Science *28.11.2007*
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Graphs & Algorithms II Exercise Set 10 HS07

URL: http://www.ti.inf.ethz.ch/ew/courses/GA07/

Homework 10

a) Let $G = (V, E)$ be a graph which does not contain $K_{s,t}$ as a subgraph, for any $s, t \in \mathbb{N}$. Prove that $\sum_{v\in V} \binom{d(v)}{s} \le (t-1)\binom{|V|}{s}$.

b) Prove that for any $s, t \in \mathbb{N}$ there is some constant $c \in \mathbb{R}$ such that $\mathrm{ex}(n, K_{s,t}) \le cn^{2-1/s}$, for any $n \in \mathbb{N}$.

Exercise 28

For some $\gamma > 0$, a graph $G = (V, E)$ is said to be γ-*far* from having some property P if one needs to change (add or remove) more than $\gamma\binom{|V|}{2}$ edges in G to make it satisfy P.

Show that for every $\gamma > 0$ there is a $\delta = \delta(\gamma)$ such that any graph $G = (V, E)$ which is γ-far from being triangle-free contains at least $\delta\binom{|V|}{3}$ triangles.

Exercise 29

a) Show that for any $n \in \mathbb{N}$ and any even g with $6 \le g \le 2\log n$ there exists a bipartite graph $G = (U \cup V, E)$ with $|U| = |V| = n$ such that $|E| \ge \frac{1}{2}n^{1+1/g}$ and G has girth at least g.
Hint: Consider a random subgraph of $K_{n,n}$ in which each edge is chosen independently with probability $n^{-1+1/g}$. Show that the expected number of cycles of length at most $g - 2$ in such a graph is at most $\frac{1}{2}n^{1+1/g}$.

b) Given any finite family $\mathcal{G}$ of graphs, prove that $\mathrm{ex}(n, \mathcal{G}) = O(n)$ if and only if $\mathcal{G}$ contains a forest. What if the finiteness condition is dropped?

Homework due: 5.12.2007, 11:00AM.