Graphs & Algorithms II　　　Exercise Set 11　　　HS07

URL: http://www.ti.inf.ethz.ch/ew/courses/GA07/

Homework 11

Show that for any $\eta \in (0,1)$ and $k \in \mathbb{N}$ there exist $\gamma = \gamma(\eta, k)$ and $\delta = \delta(\eta, k)$ with the following property.

Consider a graph $H = (\{v_1, \ldots, v_k\}, E)$ and let $V_1, \ldots, V_k$ be pairwise disjoint vertex sets of some graph G such that V_i, V_j is γ-regular for any $1 \leq i < j \leq k$. Moreover, suppose $d(V_i, V_j) \geq \eta$, if $\{v_i, v_j\} \in E$ and $d(V_i, V_j) \leq 1 - \eta$, otherwise. Then at least $\delta \prod_{i=1}^{k} |V_i|$ tuples from $V_1 \times \ldots \times V_k$ span induced copies of H.

Exercise 30

Let G be a tournament on n vertices, given as an $n \times n$ adjacency matrix A. Show that one can find the sink—if it exists—by querying $O(n)$ entries of A only.

Exercise 31

Let $\mathbb{F}$ be a finite field and $k \in \mathbb{N}$. Prove that one can test with $q = O(k + \varepsilon^{-1})$ queries (evaluations) whether a function $f : \mathbb{F} \to \mathbb{F}$ is a polynomial of degree at most k.

Exercise 32

A graph G on n vertices is ε-far from being connected if at least εn edges need to be added to G in order to make it connected.

Let G be a graph, given as an adjacency list for each vertex. Show that one can test using $O(\varepsilon^{-3})$ queries whether G is connected.

Homework due:　12.12.2007, 11:00 A.M.