

## Graphs & Algorithms II

## Exercise Set 2

## HS07

URL: <http://www.ti.inf.ethz.ch/ew/courses/GA07/>

### Exercise 4

Show that a graph is outerplanar if and only if it does not contain a subdivision of  $K_4$  or  $K_{2,3}$ .

### Exercise 5

In the *Minimum Dominating Set Problem* one is given a graph  $G = (V, E)$  and has to find a minimum cardinality subset  $D \subset V$  of the vertices such that  $D \cup N(D) \supseteq V$ .

Describe a subexponential time algorithm for Minimum Dominating Set on planar graphs.

### Exercise 6

Let  $f(n)$  be the maximum number of edges in a simple graph on  $n$  vertices that does not contain a  $K_{3,3}$  subdivision.

- For  $(n - 2) \bmod 3 \equiv 0$ , construct a graph with  $3n - 5$  edges that does not contain a  $K_{3,3}$  subdivision.
- Prove that  $f(n) = 3n - 5$  for  $(n - 2) \bmod 3 \equiv 0$  and  $f(n) = 3n - 6$ , for any other  $n \geq 2$ .  
*Hint:* Use induction and Homework 2.

### Homework 2

Prove that every 3-connected graph on at least six vertices that contains a subdivision of  $K_5$  also contains a subdivision of  $K_{3,3}$ .