

Graphs & Algorithms II

Exercise Set 4

HS07

URL: <http://www.ti.inf.ethz.ch/ew/courses/GA07/>

Exercise 10

Give a tight upper bound for the chromatic number of graphs of treewidth at most k .

Exercise 11

Prove that if $(X, T = (I, F))$ is a smooth tree decomposition of width k for a graph $G = (V, E)$ then $|I| = |V| - k$.

Exercise 12

Prove that on the $n \times n$ -grid n omniscient cops cannot catch a robber, for $n \geq 2$.

Exercise 13

Consider a graph $G = (V, E)$ and two sets $V_1, V_2 \subseteq V$ such that each vertex of V_1 is adjacent to each vertex of V_2 .

Show that in any tree decomposition $(\{X_i \mid i \in I\}, T = (I, F))$ of G there exists an $i \in I$ such that $V_1 \subseteq X_i$ or $V_2 \subseteq X_i$.

Homework 4

Describe a fixed-parameter algorithm to compute the chromatic number for graphs of bounded treewidth. That is, given a graph $G = (V, E)$ and a smooth tree decomposition $(\{X_i \mid i \in I\}, T = (I, F))$ of width at most k for G , show how to compute the chromatic number of G in time $O(f(k)p(|V|))$, where p is a polynomial and f some function (not necessarily polynomial) that depends on k only.