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# Graphs & Algorithms II — Exercise Set 7 — HS07

URL: http://www.ti.inf.ethz.ch/ew/courses/GA07/

## Exercise 19

Let $T = (V, E)$ be a directed graph on $n$ vertices, such that for any two distinct $u, v \in V(T)$ exactly one of $uv$ and $vu$ is an edge of $T$. (We say that $T$ is a *tournament*.) A vertex $x \in T$ is called a *king* of $T$ if for every vertex $y \in V$, $y \neq x$, we have $xy \in E$ or there is a vertex $z \in V$ such that $xz, zy \in E$.
Prove that $T$ has a king.

## Exercise 20

Strengthen the statement of the previous exercise. Prove that in any tournament $T$ there exists a spanning rooted "out-tree" $S \subseteq T$ of depth two, such that each vertex other than the root has outdegree at most two. (*Hint*: Create a flow network to model the desired paths to the non-successors of the root and show that every cut has enough capacity.)

## Exercise 21

Let $G$ be a graph whose odd cycles are pairwise intersecting, meaning that every two odd cycles in $G$ have a common vertex.
Prove that $\chi(G) \leq 5$.

## Exercise 22

Given a set of lines in the plane with no three meeting at a point, form a graph $G$ whose vertices are the intersections of the lines, with two vertices adjacent if they appear consecutively on one of the lines. Prove that $\chi(G) \leq 3$.

## Homework 7

Show that the performance ratio of the greedy coloring procedure could be as bad as $\frac{\Delta(G)+1}{2}$. (*Hint:* Use induction to construct, for every $k \in \mathbb{N}$, a tree $T_k$ with maximum degree $k$ and an ordering $\sigma_k$ of $V(T_k)$ such that the greedy coloring relative to $\sigma_K$ uses $k + 1$ colors.)

**Homework due:** 14.11.2007, 11:00AM.