Eidgenössische Technische Hochschule Zürich
Ecole polytechnique fédérale de Zurich
Politecnico federale di Zurigo
Swiss Federal Institute of Technology Zurich

Institute for Theoretical Computer Science *18.09.2008*
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Graphs & Algorithms II Exercise Set 1 HS08

URL: http://www.ti.inf.ethz.ch/ew/courses/GA08/

In every exercise session we provide you with an exercise sheet. Some of the exercises are to be discussed and solved in small groups during the first hour of the weekly exercise sessions.

Others - usually one or two per week - you solve in written form and return the solutions at the beginning of the subsequent exercise session. Your solutions will be thoroughly commented and graded, but they do not count towards your final grade. The motivation to work on the homework stems from your interest in the topic (and possibly also the desire to succeed on the exam :-)).

The second hour of the exercise sessions is devoted to the presentation and discussion of solutions.

Exercise 1

For a graph $G = (V, E)$, $n = |V|$, call a set $F \subseteq E$ of $f = f(n)$ edges an (f, α)-*edge-separator* if in $G \setminus F$ no component contains more than αn vertices.

a) Prove or disprove: Every tree has a $(1, \frac{1}{2})$-edge separator.

b) Prove or disprove: There is some constant $0 < \alpha < 1$ such that every tree has a $(1, \alpha)$-edge separator.

c) Show that every tree on $n \geq 2$ vertices has a $(1, \frac{\Delta-1}{\Delta} + \frac{1}{\Delta n})$-edge-separator where Δ denotes the maximum vertex degree.

Exercise 2

Show that any plane graph can be triangulated in linear time. (Resulting in a topological triangulation, not necessarily a plane straight line drawing.)

Exercise 3

Prove that there exists a constant $c > 0$ and an infinite family of planar graphs for which there is no $(c\sqrt{n}, \frac{2}{3})$-separator.

Homework 1

A graph is called outerplanar if it has an embedding in the plane such that there is some face that includes every vertex. Show that any outerplanar graph has a $(2, \frac{2}{3})$-separator.

Homework due: 25.9.2008, 11:00AM.