Exercise 35

Let $G$ be a tournament on $n$ vertices, given as an $n \times n$ adjacency matrix $A$. Show that one can find the sink—if it exists—by querying $O(n)$ entries of $A$ only.

Exercise 36

Let $F$ be a finite field and $k \in \mathbb{N}$. Prove that one can test with $q = O(k + \varepsilon^{-1})$ queries (evaluations) whether a function $f : F \to F$ is a polynomial of degree at most $k$.

Exercise 37

A graph $G$ on $n$ vertices is $\varepsilon$-far from being connected if at least $\varepsilon n$ edges need to be added to $G$ in order to make it connected.

Let $G$ be a graph, given as an adjacency list for each vertex. Show that one can test using $O(\varepsilon^{-3})$ queries whether $G$ is connected.

Homework 12

For some $\gamma > 0$, a graph $G = (V, E)$ is said to be $\gamma$-far from being triangle-free if one needs to change (add or remove) more than $\gamma \binom{|V|}{2}$ edges in $G$ to make it triangle-free.

Show that for every $\gamma > 0$ there is a $\delta = \delta(\gamma)$ such that any graph $G = (V, E)$ which is $\gamma$-far from being triangle-free contains at least $\delta \binom{|V|}{3}$ triangles.