

**Graphs & Algorithms II****Exercise Set 12****HS08**URL: <http://www.ti.inf.ethz.ch/ew/courses/GA08/>**Exercise 35**

Let  $G$  be a tournament on  $n$  vertices, given as an  $n \times n$  adjacency matrix  $A$ . Show that one can find the sink—if it exists—by querying  $O(n)$  entries of  $A$  only.

**Exercise 36**

Let  $\mathbb{F}$  be a finite field and  $k \in \mathbb{N}$ . Prove that one can test with  $q = O(k + \varepsilon^{-1})$  queries (evaluations) whether a function  $f: \mathbb{F} \rightarrow \mathbb{F}$  is a polynomial of degree at most  $k$ .

**Exercise 37**

A graph  $G$  on  $n$  vertices is  $\varepsilon$ -far from being connected if at least  $\varepsilon n$  edges need to be added to  $G$  in order to make it connected.

Let  $G$  be a graph, given as an adjacency list for each vertex. Show that one can test using  $O(\varepsilon^{-3})$  queries whether  $G$  is connected.

**Homework 12**

For some  $\gamma > 0$ , a graph  $G = (V, E)$  is said to be  $\gamma$ -far from being triangle-free if one needs to change (add or remove) more than  $\gamma \binom{|V|}{2}$  edges in  $G$  to make it triangle-free.

Show that for every  $\gamma > 0$  there is a  $\delta = \delta(\gamma)$  such that any graph  $G = (V, E)$  which is  $\gamma$ -far from being triangle-free contains at least  $\delta \binom{|V|}{3}$  triangles.