Exercise 14

Describe an algorithm that for a given graph \( G = (V, E) \) and \( k \in \mathbb{N} \) in \( f(k)p(|V|) \) time

- either returns a tree decomposition of width at most \( 4k \) for \( G \)
- or returns a \((k + 1)\)-linked set \( U \subseteq V \) of size at least \( 3k \),

where \( p(\cdot) \) is a polynomial.

*Hint.* Construct a tree decomposition \( T = ([X_i | i \in I], T = (I, F)) \) incrementally, subject to the following invariants.

1. \( T \) is a tree decomposition of width at most \( 4k \) for \( G[\cup I] \), where \( U = \bigcup_{i \in I} X_i \subseteq V \).
2. Each component \( C \) in \( G \setminus U \) has at most \( 3k \) neighbors in \( U \), and there is a single bag \( X_i \), for some \( i \in I \), that contains all of them.

Exercise 15

In a sports league, a set \( T \) of \( n \geq 2 \) teams compete for some championship. They play a large number of matches against each other; each match is between two teams and one of them wins while the other looses. At the end of season, the team(s) with most wins are champions.

At some point during the season, the management of a team wants to tell whether there is any way that they can still win the championship.

Denote by \( w(t), t \in T \), the number of games team \( t \) has won so far. And for any pair \((t_1, t_2) \in \binom{T}{2}\), let \( p(t_1, t_2) \) be the number of matches team \( t_1 \) and \( t_2 \) will still play against each other during the remainder of the season.

Based on this information, give a polynomial time algorithm to answer their question.

Homework 5

For a graph \( G = (V, E) \), a vertex cover is a set \( U \subseteq V \) of vertices such that for each edge \( e \in E \) there is at least one endpoint in \( U \), that is, \( e \cap U \neq \emptyset \).

In general, the problem of finding a minimum size vertex cover is NP-hard. Give a polynomial time algorithm for bipartite graphs.

*Homework due:* 22.10.2008, 11:00 AM.