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Graphs & Algorithms II — Exercise Set 9 — HS08

URL: http://www.ti.inf.ethz.ch/ew/courses/GA08/

Homework 9

Show that for every graph $G = (V, E)$ there is an equitable $(\Delta(G) + 1)$-coloring of its edges.

Exercise 27

Brook's Theorem says that any connected graph G that is not a clique nor an odd cycle can be colored with at most $\Delta(G)$ colors.

Find the mistake in the following "proof" of Brook's Theorem.

Induction on $n = |V|$. For $n \leq 2$ the only connected graph is a clique and there is nothing to show. For the induction step take a minimum size vertex cut $S \subset V$ (a cut exists because G is not a clique). Clearly $|S| \leq \Delta(G)$. Let $H_1, \ldots, H_k$ be the $k \geq 2$ S-lobes. By the induction hypothesis every H_i is $\Delta(G)$-colorable. Permute the labels of the colors such that they agree on the $\leq \Delta(G)$ vertices of S to obtain a proper $\Delta(G)$-coloring of G.

Exercise 28

Let $G = (V, E)$ be a bipartite graph with color classes V_1 and V_2 and $|V_1| = |V_2| = k$. Prove: If each vertex $v \in V$ has at least $\frac{k}{2}$ neighbors then G contains a perfect matching.

Exercise 29

Let $G = (V, E)$ be a tripartite graph with color classes V_1, V_2, and V_3 such that $|V_1| = |V_2| = |V_3| = k$. Prove: If each vertex $v \in V$ has at least $\frac{2k}{3}$ neighbors in each of the other two classes then G contains at least $k - 2$ pairwise vertex-disjoint triangles.

Hint: Partition V into triples consisting of one vertex from each color class. Show that ...

a) One may suppose that in every triple two chosen vertices are adjacent in G.

b) For any three triples whose vertices do not induce a triangle in G one can find a fourth triple that together with the three induces two triangles.

Homework due: 20.11.2008, 11:00AM.