

## Graphs & Algorithms II

## Exercise Set 12

## HS09

URL: <http://www.ti.inf.ethz.ch/ew/courses/GA09/>

### Exercise 32

Prove that  $K_{3,3}$  is 3-choosable.

### Exercise 33

Prove that  $K_{m,m}$  is not  $k$ -choosable for  $m = \binom{2k-1}{k}$ . Conclude that, in general, the choice number of a graph cannot be upper bounded by any function of its chromatic number.

### Exercise 34

Prove that no man can be rejected by all women in the Gale-Shapley Proposal Algorithm. Conclude that, if the algorithm terminates, it outputs a perfect matching.

### Exercise 35

Let  $C_n$ ,  $n \geq 3$  denote a simple cycle on  $n$  vertices. Prove that there exists a kernel-perfect orientation  $D$  of  $C_n$  with maximum out-degree 1 if and only if  $n$  is even. Do odd cycles have a kernel-perfect orientation as well? If so, what is the maximum out-degree in such an orientation?

### Homework 12

Prove the following claim.

**Proposition 1 (Bondy - Boppana - Siegel)** *Let  $G$  be an undirected graph and let  $D$  be a kernel-perfect orientation of  $G$ . Then  $G$  is  $f$ -choosable for  $f(v) = d_D^+(v) + 1$ .*