Exam Questions for Graph Drawing FS2010

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Basics

- Give precise definitions for the following terms:
  - graph
  - connected graph, k-connected graph
  - tree
  - matching
  - embedding
  - face
  - planar graph
  - plane graph
  - straight line drawing
- Prove that any planar graph on $n \geq 3$ vertices has at most $3n - 6$ edges and at most $2n - 4$ faces. (No need to prove Euler's Formula.)
- Prove that in any planar graph there is a vertex of degree at most five.
- What is the statement of Kuratowski's Theorem? (not the proof!)

Planarity Testing

- What is an LR partition? In particular, this requires you to explain what back edges and lowpoints are.
- Why does every plannar graph have an LR partition?
- Outline the main steps in the proof that a graph is planar if it has an LR partition!

Convex Drawings

- Give a precise definition for *convex drawing*.
- Give a complete proof for Fary's Theorem.
- What is the statement of Tutte's Spring Theorem?
- Sketch the inductive proof of Tutte's Theorem due to Thomassen: Explain the interpretation in terms of energy function and springs, and the inductive step using Thomassen's Lemma (Claim 2 on the slides). You do not need to prove existence and uniqueness of the minimum energy drawing (Claim 1).
- Prove Thomassen's Lemma (linked in Week #6 on the webpage).

Crossing Lemma

- What is the crossing number of a graph? Give a precise definition!
- What is the statement of the Crossing Lemma?
- Give a proof of the Crossing Lemma!
- What is a point-line incidence?
- Sketch how the Crossing Lemma can be used to prove an upper bound on the number of point-line incidences between n points and m lines in the plane! What is the resulting upper bound?
- Sketch how the Crossing Lemma can be used to prove an upper bound on the number of unit distances among a set of n points in the plane! What is the resulting upper bound?

Grid Drawings

- Give a precise definition for *canonical order* (you should be able to define and explain all terms that occur as part of this definition).
- Which graphs admit a canonical order? Give a detailed proof including time complexity.
- Describe in full detail the shift algorithm by de Fraysseix et al; including the linear time implementation due to Chrobak and Payne.

Drawing Trees

- Which criteria should a drawing of a rooted binary tree ideally satisfy? Describe the "wish-list" discussed in the lecture.
- Describe in full detail the algorithm by Reingold and Tilford to draw rooted binary trees according to this wish-list. Which kind of guarantees does this algorithm provide regarding its output drawing?

- Sketch how to model the tree drawing problem as a linear program.

Map Labeling

- What is the point labeling problem (problem R4 from the lecture)?
- What can be said about the computational complexity of the point labeling problem?
- Sketch an efficient algorithm for approximating the optimal label size in the point labeling problem! What is the approximation factor, and how is it obtained?

Rectangular Drawings

- What is a rectangular drawing of a graph?
- Which graphs admit a rectangular drawing, which don't? Give a few examples...
- Describe in full detail the algorithm to decide in $O(n^{3/2})$ time whether a given plane graph on n vertices admits a rectangular drawing.

Orthogonal Drawings

- What is an orthogonal drawing of a graph?
- Which graphs have orthogonal drawings? Justify your answer!
- Sketch an algorithm to find an orthogonal drawing equivalent to a given drawing, with a minimum number of bends. In particular, you should be able to define (a) the min-cost flow problem in the setting of general graphs, (b) what an orthogonal representation of a graph is.

Layered Drawings

- What is a layered drawing of a directed graph?
- What are desirable quality criteria of a layered drawing?
- Sketch an algorithm to obtain a good layered drawing in practice!
- Comment on the theoretical aspects of the problem and describe at least two NP-hard subproblems that come up during the construction of a goof layered drawing.