Exercise 1 (Exercise 2 from Set 4, question (b) is not part of the oral exam)

(a) Show that if a point set $P \subseteq \mathbb{R}^2$ is $n$-universal, then it is also $k$-universal, for any $1 \leq k \leq n$.

(b) Show that if there exists an $n$-universal point set $P \subseteq \mathbb{R}^2$ of $m$ points, then there exists an $(n-1)$-universal set of $m-1$ points.

Exercise 2

Recall the explicit construction argument from the lecture that we used to prove that there is a collection of 7'393 planar graphs on 35 vertices that do not admit a simultaneous plane straight-line embedding without mapping.

The argument uses a set of seven graphs with fixed outer face that are mapped onto a bipyramid skeleton (Figure 3.4 in the lecture notes). Can you give a similar bound on the number of planar graphs that admit a simultaneous plane straight-line embedding without mapping if you

- use a tetrahedron skeleton instead of a bipyramid and
- use the set of three graphs with fixed outer face from Figure 3.3 in the lecture notes instead of the seven graphs with fixed outer face from Figure 3.4 in the lecture notes?