Exercise 1

(a) Prove: every SEG-graph has a representation where all segment endpoints are integral.

(b) Prove: the problem of deciding whether or not a graph has a 100-STRING representation is in NP.

Exercise 2

Let \( U \subseteq \mathbb{R}^2 \) be an open arcwise connected set; that is, every two points of \( U \) can be connected by a simple curve. Prove, as rigorously as possible, that every two points of \( U \) can also be connected by a polygonal curve.

Exercise 3

Recall that we call a weak realization standard if the corresponding drawing of \( G \) is standard, by which we mean that the edges are drawn as polygonal curves, every two intersect at finitely many points, and no three edges have a common intersection (where sharing a vertex does not count).

(a) Prove that if \((G, R)\) has a weak realization, then it also has a standard weak realization.

(b) Prove that if \((G, R)\) has a weak realization \( W \) with finitely many edge intersections in which no three edges have a common intersection, then it also has a standard weak realization \( W' \) with at most as many edge intersections as in \( W \).

Exercise 4

Given graphs \( G_1 = (V, E_1), \ldots, (V, E_k) \), simultaneous geometric embedding with mapping (k-SGE) is the problem of finding a set \( P \) of \( |V| \) points in the plane and a bijection \( \chi : V \to P \) such that \( \chi \) is a crossing-free straight-line drawing for all \( G_i \).

Given an abstract topological graph \((G, R)\), a weak straight-line realization is a straight-line drawing of \( G \) where all crossing pairs are in \( R \).

Prove that k-SGE is polynomially equivalent to the problem of finding a weak straight-line realization.