About the homework sets

This is the last of three sets of homework exercises. You have two weeks to solve this set and send it, typeset in LaTeX and as a PDF, to htyagi@inf.ethz.ch. Your three grades each count for 10% of your final grade. Unless explicitly mentioned otherwise, all your algorithms must be accompanied by a correctness proof and a runtime analysis. It is not allowed to solve the exercises in groups. If you use any material beyond what was proven or referenced to in the lecture, you must provide adequate references.

To create illustrations, we strongly recommend Ipe (http://ipe7.sourceforge.net/). It can create PDF figures that include text/labels using the original \LaTeX\ fonts and symbols.

Homework 1 (20 points)

Describe and analyze an algorithm for computing the convex hull of \( n \) random points in the plane. By random points, we mean that each of the \( n \) points is chosen independently and uniformly at random from either the unit square, or the unit disk.

Your algorithm should in both cases have expected runtime \( O(n) \), where the expectation is over the random choice of the input point set and (possibly) internal random choices of the algorithm.

Hint: Use randomized incremental construction! You may assume the following results (see eg. http://arxiv.org/abs/1111.5340): The convex hull of \( n \) points chosen independently and uniformly at random has an expected number of

1. \( O(\log n) \) vertices, if the points are chosen from the unit square, and
2. \( O(n^{1/3}) \) vertices, if the points are chosen from the unit disk.

Homework 2 (15 points)

We are given two sets of points in \( \mathbb{R}^2 \), one set representing cancer cells, the other one representing healthy cells. Let \( n_1 \) denote the number of cancer cells and \( n_2 \) denote the number of healthy cells. You can assume that the set of all (cancer and healthy) cells is in general position. Now we want to destroy the maximum number of cancer cells without damaging any healthy cells. Show how to compute an open disk that contains as many cancer cells as possible but no healthy cell (cells on the boundary of the disk are not counted either way), in time \( O(n_2 \max\{\log(n_2), n_1 \log(n_1)\}) \).
Homework 3 (15 points)

Describe an $O(n^2)$ time algorithm that given a set $P$ of $n$ points in the plane finds a sub-
set of five points that form a strictly convex empty pentagon (or reports that there is none if
that is the case). Empty means that the convex pentagon may not contain any other points of $P$.

Hint: For each $p \in P$ discard all points left of $p$ and consider the polygon $S(p)$ formed by $p$ and
the remaining points taken in circular order around $p$. Explain why it suffices to check for all
$p$ whether $S(p)$ has 4 vertices other than $p$ that form an empty convex quadrilateral. How do
you check this in $O(n^2)$ time?