Skript-Aufgabe 30 (4 Punkte)

Show that the following sets of functions are complete for the set of binary Boolean functions.

b) \{OR, NOT\}

c) \{NAND\}

d) \{NOR\}, where NOR := NOT \circ OR.

e) \{XOR, AND\}

You may use the fact that \(\{AND, OR, NOT\}\) is a complete set of binary Boolean functions.

Skript-Aufgabe 32 & 33 (4 Punkte)

Parenthesize the following expressions according to operator precedences and associativities. Then, evaluate the expressions step-by-step, assuming that \(x, y,\) and \(z\) are all of type \texttt{int} with \(x=0, y=1,\) and \(z=2.\)

b) \(z > 1 \land x != 2 - 2 == 1 \land y\)

c) \(3 * z > z || 1 / x != 0 \land 3 + 4 >= 7\)

Skript-Aufgabe 49 (4 Punkte)

Write a program \texttt{perfect.cpp} to test whether a given natural number \(n\) is perfect. A number \(n \in \mathbb{N}\) is called \textit{perfect} if and only if it is equal to the sum of its proper divisors, that is, \(n = \sum_{k \in \mathbb{N}, \text{ s.t. } k < n \land k \mid n} k.\) For example, \(28 = 1+2+4+7+14\) is perfect, while \(12 < 1+2+3+4+6\) is not.

Extend the program to find all perfect numbers between 1 and \(n.\) How many perfect numbers exist in the range \([1, 50000]?\)
Skript-Aufgabe 52 (4 Punkte)

We heard in the lecture that it took Frank Nelson Cole around three years to find the factorization
\[ 761838257287 \times 193707721 \]
of the Mersenne number \( 2^{67} - 1 \) by hand calculations. Write a program `cole.cpp` that performs the same task (hopefully in less than three years).

Hint: You will need the type `ifm::integer`.

Die Aufgaben 36 und 55 aus den Vorlesungsunterlagen sind Challenge Aufgaben und geben jeweils 8 Punkte, wenn sie vollständig gelöst werden.

Abgabe: Bis 1. November 2011, 15.15 Uhr.