Informatik für Mathematiker und Physiker  HS12

Exercise Sheet 4

Submission deadline:  3.15pm - Tuesday 16th October, 2012

Course URL: http://www.ti.inf.ethz.ch/ew/Lehre/Info1_12/

Note: For the tasks Assignment 1 and Challenge 1, you are not allowed to use any other libraries than <iostream> and <IFM/Integer.h> and only the commands you have seen in the lectures until now.

Assignment 1 - (2 points)

The integer square root of a given positive integer $n$ is defined as $\lfloor \sqrt{n} \rfloor$. The <IFM/Integer.h> library does not provide a method for getting the integer square root for a given number.

Write a program integerSqrt.cpp that reads from the standard input a non-negative number $n$ of the type ifm::integer and outputs its integer square root.

Challenge 1 - (4 points)

Write a program integerSqrtChallenge.cpp that reads from the standard input a non-negative number $n$ of the type ifm::integer and outputs its integer square root. The program should be capable of calculating the integer square root of a 100-digit integer within 1 second.

Assignment 2 - (4 points)

Write a program perfect.cpp to test whether a given natural number $n$ is perfect. A number $n \in \mathbb{N}$ is called perfect if and only if it is equal to the sum of its proper divisors, that is, $n = \sum_{k \in \mathbb{N}, s.t. k < n \land k \mid n} k$. For example, $28 = 1 + 2 + 4 + 7 + 14$ is perfect, while $12 < 1 + 2 + 3 + 4 + 6$ is not.

Extend the program to find all perfect numbers between 1 and $n$. How many perfect numbers exist in the range $[1, 50000]$?
Assignment 3 - (5 points)

The number $\pi$ can be defined through various infinite sums. Here are two of them.

\[
\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots \\
\frac{\pi}{2} = 1 + \frac{1}{3} + \frac{1}{3 \cdot 5} + \frac{1}{3 \cdot 5 \cdot 7} + \cdots
\]

Write a program for computing an approximation of $\pi$, based on these formulas. Which formula is better for that purpose?

Assignment 4 - (5 points)

Noah is the owner of a well-established shop in Wengen and has his regular customers. He estimates that with his regular customers, he makes $m$ CHF net profit a year. He wants to put his profit into Wengen Dorf Bank, that promises $p\%$ interest on the money on his account every year. Noah wants to know how much money he would have on his account after $n$ years, if he deposits $m$ CHF every year onto the account under $p\%$ interest, assuming that at the beginning, the account was empty.

Write a program interests.cpp that reads $m, n$ and $p$ from the standard input and outputs the amount of money that is at Noah’s account after he deposits $m$ CHF for $n$ years on the account with $p\%$ interest rate.

The example inputs and outputs are:

The yearly amount $m$ =? 1
The yearly interest (in %) $p$ =? 5
The number of years $n$ =? 1
The total amount on the account is 1.05

The yearly amount $m$ =? 1
The yearly interest (in %) $p$ =? 5
The number of years $n$ =? 2
The total amount on the account is 2.1525

Challenge 2 - (4 points)

The largest Mersenne prime known as of September 2009 is

\[2^{43,112,609} - 1\]

Write a program famous_last_digits.cpp that computes and outputs the last 10 decimal digits of the above Mersenne prime!