

Ecole polytechnique fédérale de Zurich Politecnico federale di Zurigo Swiss Federal Institute of Technology Zurich

Institut für Theoretische Informatik Emo Welzl, Jiří Matoušek 17.06.2004

## Theoretische Informatik (Kernfach) SS 2004 Exercise Set 12

## Exercise 1

Let S be a set of  $n \ge 2$  non-crossing segments in the plane. Show that the set P(S) of endpoints of S satisfies

$$2 + \frac{n}{3} \le |P(S)| \le 2n \; .$$

## Exercise 2

Let S be a nonempty set of non-crossing segments in general position in the plane, let  $s \in S$ , and let T be a trapezoid in  $\mathcal{T}(S \setminus \{s\}) \setminus \mathcal{T}(S)$ . Depending on the number of endpoints of s inside T (0, 1, or 2), investigate how many trapezoids overlapping with T can be created by adding s to  $S \setminus \{s\}$ .

## Exercise 3

We are given a set P of n points in  $\mathbb{R}^2$  and a point q which has distinct distances to all points in P. We add the points of P in random order (starting with the empty set), and observe the nearest neighbor of q in the set of points inserted so far. What is the expected number of distinct nearest neighbors that appear during the process?